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Book Review: Nonlinear Phenomena in Physics and Biology

Nonlinear Phenomena in Physics and Biology.
Edited by R. H. Enns, B. L. Jones, R. M. Miura, and S. S. Rangnekar, Plenum Press, New York, 1981.

This volume succeeds in synthesizing a number of unrelated topics from physics and biology by means of the underlying structure of the nonlinear equations of evolution. The physical concepts that have evolved through the solution of certain systems of nonlinear differential equations in physics, e.g., solitons and dissipative structures, have been emphasized. Individual lectures are often heavily weighted towards the specialty of the particular investigator; however, the volume as a whole has managed to remain fairly well balanced among the three areas of physics, biology, and mathematics. I would recommend this book to physicists and mathematicians interested in learning the types of biological problems that have yielded, at least in part, to their familiar methods, and to the biologist who is interested in learning some of the physical and mathematical concepts that may assist his understanding of biological processes.

The first lecture by A. C. Scott, on an "Introduction to Nonlinear Waves," is a broad-based historical review of solitary wave motion. The review starts with the discovery of the solitary wave by Russell in 1834 and traces its development up through the nonlinear field theories of the 1920s and 1930s and includes the more recent soliton solutions of the Kortweg–deVries (KdV), sine-Gordon, and nonlinear Schrödinger equations and the Toda lattice. The reviews are brief, but of sufficient clarity to provide the physical context in which the *bimolecular dynamic equations* of Davydov can be introduced. The transition of the soliton concept from applications in physics to those in biology is made in a relatively painless way. The paper emphasizes a number of historical priorities in the development of the concept of a soliton and points out that its importance in biological systems may be as great as or greater than in physics systems.

The lecture "Remarks on Nonlinear Evolution Equations and the Inverse Scattering Transform" by M. J. Ablowitz is, in large part, a

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compendium of continuum and discrete equations having soliton and/or inverse scattering transform solutions. The recent work on the intermediate, long-wave equation which has the KdV and Benjamin–Ono equations as limiting forms is reviewed.

D. J. Kaup focuses on the basics of three-dimensional inverse scattering, such as for systems with three-wave resonant interactions (3WRI) in three dimensions in "The Linearity of Nonlinear Soliton Equations and the Three-Wave Resonant Interaction." The formalism of 3WRI is adopted to the scattering of a wave envelope from a localized potential yielding a transmitted wave and two scattered waves with the corresponding transmission and reflection coefficients. One finds in analogy to one-dimensional solitons obtained in one-dimensional inverse scattering that there exist three-dimensional "lumped solitons" through the use of Backlund transformations. The principal difference between one- and three-dimensional inverse scattering is the lack of bound states in the latter case, so that no 3D solitons exist. This paper outlines an important segment of recent research for investigators interested in the inverse problem.

In the lecture "Contour Dynamics: A Boundary Integral Evolutionary Method for Inviscid Incompressible Flows," N.J. Zabusky presents an algorithm for calculating a two-dimensional inviscid flow field. The algorithm enables one to calculate the strong (but finite) interaction between vortex states in a fluid, i.e., rotating and translating stationary solutions of the 2D hydrodynamic equations, which are natural extensions of the motion of a point vortex. The internal degrees of freedom in this "V-state" lead to fluid motions quite different from those observed in calculations using point vortices. The author concludes that the calculations strongly suggest that point-vortex algorithms are inadequate for determining fine-scale fluid motion if there are many close encounters. This method provides the first 2D stationary solution to the MHD equations.

B. Fornberg discusses the "Numerical Computation of Nonlinear Waves" and reviews the major computation methods for solving the Korteweg–deVries equation and those for deep water waves in one horizontal dimension.

G. Nicolis provides a general review of the nonequilibrium thermodynamic concept of a dissipative structure in his lecture "Bifurcations, Fluctuations and Dissipative Structure." The general arguments concerning the compatibility of the concepts of dissipation, reversibility, and order are presented in a way so as to contrast the idea of order resulting from steady fluxes in dissipative systems and order resulting from conservation laws and Hamiltonian dynamics (as in a soliton). A discussion of the stability properties of nonlinear systems is presented in the language of bifurcation analysis and is illustrated using various models of chemical reactions; in particular, reaction-diffusion equations are discussed in some detail. The master equation formalism is also reviewed to provide an understanding of thermodynamic fluctuations and the process of fluctuation-induced phase transitions. The deep, and largely unexpected, relationship between bifurcation phenomena and stochastic processes is then discussed.

The lecture "Chemical Oscillations" by L. N. Howard concentrates on techniques for solving nonlinear rate equations in systems with more than one degree of freedom. In particular, the problems of chemical oscillations and nonlinear waves are examined using bifurcation theory and singular perturbation methods. The Belousov–Zhabatinskii reaction, along with their reaction–diffusion equations are examined.

The first truly mathematical biology lecture is that of J. Rinzel on "Models of Neurobiology." He is concerned with mathematical models of neural communication and restricts much of his discussion to the modification in cell membrane properties due to applied stimulating currents. The biophysical distinction between repetitive neural firing and bursting in spatially homogeneous membranes is discussed. This latter effect is described by two alternative models: the first proposes a mechanism within an individual cell, whereas the second assumes a cooperative mechanism. For spatially inhomogeneous membranes, models of pulse trains are examined.

R. M. Miura provides a brief description of mathematical modeling strategies by way of introducing the topic "Nonlinear Waves in Neuronal Cortical Structures." The body of the paper is concerned with describing the phenomenon of "spreading depression" as a nonlinear wave propagating in a population of neurons. The biology and chemistry associated with the depressed neural activity is reviewed as are previously proposed theories. Solutions to certain reaction–diffusion equations are shown to yield some of the observed qualitative features of spreading depression.

In the two papers on "Bifurcation in Insect Morphogenesis," S. A. Kauffman concentrates on the problems of pattern formation in developmental biology. The biological discussion focuses on the fruit fly, *Drosophila melanogaster*, about which a great deal is known in connection with duplication, regeneration, and pattern regulation in general. Positional information is necessary for pattern regulation and it is proposed that chemical concentration gradients spanning a tissue, whose concentration levels specify the positional information of cells at each point in a domain, is the physical mechanism. A system of reaction–diffusion equations describes this mechanism, and they are analyzed using bifurcation theory to determine the positional information on the wing section of a fruit fly with respect to one- and two-positional axis. The mechanism appears viable because biochemical reaction–diffusion systems generate transverse gradients in growing asymmetrical tissue domains. The second lecture is a somewhat more qualitative discussion of the ways in which cells in different regions of the embryo of a fruit fly come to adopt different development programs. The bifurcation studies of a set of reaction–diffusion equations such as used above are discussed in this context.

P. Schuster is concerned with the problem of self-replication and, in particular, self-organization through self-replication in the lecture "Selection and Evolution in Molecular Systems." The emphasis is on molecular systems using chemical kinetics. Various models of autocataytic reactions are discussed as models of RNA-replication and the formation of n-dimension hypercycles. Also discussed are the effects of errors on the replication process, i.e., stochastic kinetic equations for macromolecular self-organization.

The lecture of D. Ludwig on "Escape from Domains of Attractions for Systems Perturbed by Noise" is an application of some of the techniques used in solving nonlinear stochastic differential equations to a problem in evolution. The emphasis is on the analysis of exit time statistics in both single-degree and many-degree of freedom systems. A limitation of the techniques is the delta-correlated noise in time and the consequent use of the Ito calculus in the discussion.

Bruce J. West
Associate Director
Center for Studies of
Nonlinear Dynamics
La Jolla Institute
POB 1434
La Jolla, Cal. 92038